This time we discussed the first derivative test. It applies in more general situations than the second derivative test. Its only disadvantage is that it is sometimes a little more work to use.

Suppose we have a function $f(x)$ with a critical point at $x = a$, meaning $f'(a) = 0$. The **first derivative test** is a way to determine whether a local minimum or maximum or neither occurs at the point.

The idea is simple: if there is a local maximum, then $f(x)$ must be increasing at values of x on the immediate left of $x = a$, and then decreasing afterwards - if it were decreasing before or increasing after, then $f(a)$ couldn't be the largest nearby value! The opposite holds for a local minimum.

As we pointed out last time, whether $f(x)$ is increasing or decreasing can be determined from the derivative: where $f'(x)$ is positive, $f(x)$ is increasing, and where $f'(x)$ is negative, $f(x)$ is decreasing.

So the first derivative test simply says: if $f'(x)$ changes sign from positive to negative when going from left to right near $x = a$, then the point is a local maximum. If instead $f'(x)$ changes signs from negative to positive, then there is a local minimum. If the sign does not change, then $x = a$ is neither a local maximum nor local minimum.

Let's summarize the process of optimization. An optimization question is where you have a function $f(x)$ and want to identify its maxes and mins.

1. Determine all possible locations for minima and maxima. There are two types. Critical points, which you find by solving $f'(x) = 0$; and the left and right endpoints of the domain of $f(x)$, if there are any.

2. For each of the critical points, use the first or second derivative test to determine which are local maxima or minima or possibly neither (remember: if the 2nd derivative test is inconclusive, you should try the first derivative test).

3. Plug in all of the possible locations of minima and maxima, then compare the values you get to determine the global minima and maxima.

In some ways, this is mostly an algebra problem: the difficult part is finding the critical points, i.e. the zeros of $f'(x)$. It can also be difficult to extract the domain of the function from word problems: the main thing to keep in mind is that physical quantities (length, weight, temperature in kelvin, etc) can't be negative, so any point which is not physically achievable isn't in the domain.