A function is a rule that takes in an input (like a number) and produces some output (usually another number).

For instance, $f(x) = x^2 + 2x + 1$ is a function. The input is a number x, and the output is the value of $x^2 + 2x + 1$. For instance, $f(2) = 2^2 + 2 * 2 + 1 = 9$. In fact, this is an example of a quadratic function, which is a special kind of polynomial. These are very important to this course.

You could think of an animal as a function that takes food as input, and outputs the behavior of that animal!! (e.g. a rabbit eats hay and turns it into a heartbeat, breathing, hopping, some waste, etc).

Most commonly in sciences, functions describe a relationship between two interesting numerical quantities. In physics, you might be interested in speed as a function of time. In chemistry, you might be interested in the concentration of a molecule over time.

Some functions we will encounter in this course are: linear, quadratic, polynomials, square root, and piecewise functions. Linear and quadratic functions are actually special kinds of polynomials, but they're also of independent interest. Later on, we will learn about exponential, logarithmic, and trigonometric functions (you don't need to know these now!).

It is important to note that functions don't all have to be called $f(x)$ – the names "f" and "x" can be changed. For instance, when talking about velocity as a function of time, we might write $v(t)$ so that the letters remind us of the quantities they represent.

[some textbooks rigidly stick to x as the input and y as the output]

Here are descriptions of some important functions. For pictures, see the worksheet solutions.

Linear: A linear function is one that can be written in the form $f(x) = mx + b$, for constants m and b. For instance, $f(x) = 2x + 1$ or $f(x) = -3x + 5$ are both linear functions corresponding to $m = 2, b = 1$ and $m = -3, b = 5$, respectively. When graphed, these functions look like a line.

Quadratic: A linear function is one that can be written in the form $f(x) = ax^2 + bx + c$, for constants a, b, c, where we require that a is not zero (if $a = 0$, then we end up with a linear function!). For instance, $f(x) = x^2$ and $f(x) = x^2 - 2x - 3$ are both quadratic functions. When graphed, they make an upward- or downward-facing "U" shape.

Polynomial: Both linear and quadratic functions are special kinds of polynomials. A polynomial is, roughly, any function that can be formed only by using addition and multiplication. The following are all polynomials:

$$f(x) = 2x$$
$$f(x) = x^3 - 3x + 1$$
$$f(x) = x + x - 3x + 5xx^2 + 1$$
$$f(x) = x^{55} - x^3 + 1$$
$$f(x) = (x^2 + 3x + 1)(x^4 - 3)$$

Every polynomial can be "expanded" into an expression only involving whole number powers of x (meaning things like x^3, x, x^8, but not x^{-1} or $x^{2.1}$ or $x^{1/2}$) multiplied by some numbers and added together. In this form, we say that the "degree" is the highest power of x that appears in the polynomial. For instance, quadratics all have degree 2.

In the examples above, the degrees are, in order, $1, 3, 3, 55, 6$.

Square root: This function is our first that's not a polynomial. The square root of x, denoted $\sqrt{x}$, is defined to be the number that squares to x. Since the square of a number is always positive, this means that the square root can only handle positive inputs.

Absolute value: This function is also not a polynomial. The absolute value of x is written $|x|$. It takes in the number x and makes it positive. For instance $|5| = 5$ doesn't change x because it is already positive, while $|-2| = 2$, changing from negative to positive two.

Piecewise functions: These are our "strangest" functions. A piecewise function is any function that looks like a combination of other functions tossed together. The only examples we know so far are for calibration functions.

Remember that for oddly shaped bottles – such as the "ink bottle" which went from straight to sloped sides, the graph has two different regions, one corresponding to the calibration of a straight-side bottle, and another to the calibration function of a slanted-side bottle. These different "pieces" are why such functions are called piecewise.

Another piecewise function is the absolute value: for negative inputs x, the absolute value returns the line $y = -x$, while for positive inputs x, it just gives $y = x$. So it is a piecewise function formed of two lines.