

1. Evaluate each of the following polynomials at 0.1 without “doing any work”.

(a) $4 + x + x^2 + 7x^4$ 4.1107

(b) $x^2 + 4x^3$ 0.014

(c) $236 + 9x^4 + 2x^5 + 2x^6 + 2x^7$ 236.00092522

2. Evaluate each of the following power series at 0.1.

(a) $1 + x + x^2 + x^3 + x^4 + \dots$ 1.1111...

(b) $x + 2x^2 + x^3 + 2x^4 + x^5 + \dots$ 0.12121....

(c) $2 + x^2 + x^4 + x^6 + x^8 + \dots$ 2.010101....

3. For each of the following decimal numbers, write a polynomial or power series that evaluates to them at 0.1.

(a) 1.22 $1 + 2x + 2x^2$

(b) 38.040404... $38 + 4x^2 + 4x^4 + \dots$

(c) 5.72527252725... $5 + 7x + 2x^2 + 5x^3 + 2x^4 + 7x^5 + \dots$

4. It is a fact that for sufficiently small x , the function $f(x) = \sqrt{x+1}$ is equal to the power series

$$1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} + \dots$$

(calculator recommended for this problem)

(a) Using the degree 1 truncation of this power series, find an approximation for $f(1) = \sqrt{2}$. Is this close to the actual value of $\sqrt{2}$?

$f(x) \approx 1 + \frac{x}{2} \rightarrow f(1) \approx 1 + \frac{1}{2} = 1.5$
 near to $\sqrt{2} \approx 1.41$

(b) Repeat part (a) with the degree 3 truncation instead. Compare the two: is one approximation better than another? Why might that be?

$f(1) \approx 1 + \frac{1}{2} - \frac{1}{8} + \frac{1}{16} = 1.4125$

(c) Again using the degree 3 truncation of this power series, find an approximation for $f(4) = \sqrt{5}$. Is this close to the actual value of $\sqrt{5}$?

$f(4) \approx 1 + \frac{4}{2} - \frac{4^2}{8} + \frac{4^3}{16} = 5$. Not even close

5. Thinking problems. Do not worry about correct answers or spend lots of time on these problems (they’re just puzzles to have in mind before the next class). But do try to write a little for each part.

(a) Could you evaluate the following power series at $x = 0.1, 3, 20$?

$$1 + x + 2x^2 + 3x^3 + 4x^4 + \dots$$

yes, no, no
 \rightarrow too big

\uparrow
 much closer
 (more terms)

(b) Could you evaluate the following power series at $x = -1$?

$$1 - x + x^2 - x^3 + x^4 - x^5 + \dots$$

no $\rightarrow 1 + (-1) + (-1)^2 + (-1)^3 + \dots$
 $= 1 + (-1) + 1 + (-1) + \dots$
 not a number