1. Calculate derivatives of the following functions

(a) $f(x) = x^4 - x + 5$

$4x^3 - 1$

(b) $f(x) = (x^{200} + x^{100} - 1)x^2$

$(200x^{199} + 100x^{99})x^2 + (x^{200} + x^{100} - 1)2x$

(c) $f(x) = (x^2 - x + 1)e^x$

$(2x-1)e^x + (x^2 - x + 1)e^x$

(d) $f(x) = 3e^x + e^{2x} - x$

$3e^x + 2e^{2x} - 1$

(e) $f(x) = (e^x + 3x)(x^2 - x + 2)$

$(e^x + 3)(x^2 - x + 2) + (e^x + 3x)(2x - 1)$

(f) $f(x) = e^{4x^2+x}$

$(8x + 1)e^{4x^2+1}$

(g) $f(x) = (x^3 - 3x + 1)(x + x + 5x^2)$

$(3x^2 - 3)(2x + 5x^2) + (x^3 - 3x + 1)(2 + 10x)$

(h) $f(x) = e^{2x+3}e^{3x+2}$

$2e^{2x+3}e^{3x+2} + 3e^{2x+3}e^{3x+2}$

2. Expand the derivatives of the following using the derivative rules:

(a) $f(g(x))h(x)$

$f'(g(x))\,g'(x)\,h(x) + f(g(x))\,h'(x)$

(b) $f(g(x)+h(x))$

$f'(g(x)+h(x))\left(g'(x)+h'(x)\right)$

(c) $(f(x)+g(x))(h(x))$

$\left(f'(x)+g'(x)\right)h(x) + \left(f(x)+g(x)\right)h'(x)$

(d) $(f(x))^2+g(x)$

$2f(x)f'(x) + g'(x)$

3. Suppose you know that $f(0)=1$ and $f'(0)=-2$. Calculate $g'(0)$ for the following functions g:

(a) $g(x)=f(x)^3-f(x)+3x$

$g'(x) = 3f(x)^2 f'(x) - f'(x) + 3$

at zero $3\cdot 1^2\cdot -2 - (-2) + 3 = -6+5 = -1$

(b) $g(x)=e^x f(x)$

$g'(x) = e^x f(x) + e^x f'(x)$

at zero $1\cdot 1 + 1\cdot(-2) = -1$

(c) $g(x)=f(x)-(e^x)^2$

$g'(x) = f'(x) + 2e^{2x}$

at $x=0$, $-2 + 2\cdot 1 = 0$

(d) $g(x)=f(x)^5-e^x f(x)$

$g'(x) = 5f(x)^4 f'(x) - e^x f(x) - e^x f'(x)$

at zero $5\cdot 1^4(-2) - 1\cdot 1 - 1\cdot(-2) = -10-1+2$

$= -9$

4. A moss farm is trying to improve its efficiency. They know that moss grows at a rate of 2 square feet per 50 square feet of moss per hour (quite slowly). Ideally, the moss should be harvested as soon as it covers 4000 square feet.

Write a rate equation describing the growth of the moss, solve it, and then using a calculator estimate the best time to harvest.

$$\begin{cases} M'(t) = \frac{2}{50} M(t) \\ M(0) = \text{missing - oops!} \end{cases}$$

continuously?

5. You are seeking a \$15000 loan to renovate your kitchen. A bank offers you two choices: you can pay 10% interest, compounded [redacted] for one year, OR you can pay 6% interest, but only compounded twice (in other words, you would have a total balance of $15000 \times 1.06 \times 1.06 = 16854$). Write a rate equation describing the first option, solve it, and then decide which of the two loan options you prefer.

$$\begin{cases} B'(t) = .10B(t) \\ B(0) = 15000 \end{cases} \rightsquigarrow 15000 e^{t/10} = B(t)$$

at $t = 1$, $B(1) = 15000e^{1/10} = 16577.56$

cheaper!