1. Determine the derivatives of the following functions.

(a) $f(x) = x^3e^x + 2x^2 + e^x - 1$ (prod)

$3x^2e^x + x^3e^x + 4x + e^x$

(b) $f(x) = e^x + 3e^{2x}$ (comp)

$e^x + 6e^{2x}$

outside $= e^x \rightarrow e^x$

inside $= 2x \rightarrow 2$

(c) $f(x) = (e^x + 1)(x^2 + 3x - 2)$ (prod)

$e^x(x^2+3x-2) + (e^x+1)(2x+3)$

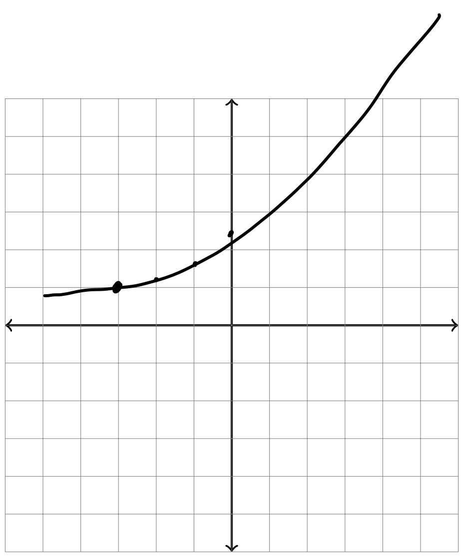
(d) $f(x) = xe^x$ (prod)

$e^x + xe^x$

2. Sketch a solution to the following rate equation:

$$f'(x) = \frac{1}{4}f(x)$$

$$f(-3) = 1$$



x	f	f'
-3	1	1/4
-2	1.25	5/16
-1	⋮	⋮

3. Suppose a population 40 ferrets is introduced to a forest. Assume the ferrets produce 1 baby ferret per month per 10 ferrets. Write a rate equation describing the ferret population function, then give an exact solution to it. Using the exact solution and a calculator, estimate when the forest will have more than $100,000$ ferrets.

$$\left.\begin{aligned} P'(t) &= \tfrac{1}{10} P(t) \\ P(0) &= 40 \end{aligned}\right\} \qquad 40\, e^{t/10} = P(t)$$

t months

4. Some foods, like kimchi and sauerkraut are fermented. This involves encouraging the growth of certain bacteria which release chemicals and process the food. In practice, once the population of bacteria is large enough, they will all die off (because of the waste they produce). To efficiently run a fermentation factory, it is important to know when this will happen.

5. Suppose that jars are inoculated with 1000 fermenting bacteria, and that on average it takes one bacterium an hour to grow and triple into 3 bacteria. Write and solve a rate equation for this situation. If the bacterial population dies off after the population reaches $500,000,000$ bacteria, approximately how many hours will it take for the jar to be free of bacteria. (note: not realistic numbers)

$$\left.\begin{aligned} P'(t) &= 2P(t) \\ P(0) &= 1000 \end{aligned}\right\} \qquad P(t) = 1000\, e^{2t}$$

t hours

6. Some modern waste treatments are quite similar to fermentation: bacteria are introduced to waste water in order to consume toxic chemicals.

Suppose we are treating waste water with a kind of bacteria that takes roughly 10 hours to reproduce into 2 more bacteria. A waste pond requires a population of at least $200,000,000,000$ bacteria to function well. If we start with only 2000 bacteria, how long will it take a waste pond to be operable? What if we started with 5000?

$$\left.\begin{aligned} P'(t) &= P(t) \\ P(0) &= 2000 \end{aligned}\right\} \qquad P(t) = 2000\, e^{t}$$

t 10 hours