Here are our fundamental rules for taking derivatives. With them, we can calculate the derivative of ***any*** function that's built out of functions whose derivatives we already know.

Rule	Input Function	Derivative
Sum	$f(x) + g(x)$	$f'(x) + g'(x)$
Product	$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
Quotient	$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$
Chain	$f(g(x))$	$f'(g(x))g'(x)$

There are two kinds of functions we currently know how to differentiate: polynomials, and the exponential function:

Rule	Input Function	Derivative
Power	x^n	nx^{n-1}
Exponential	$e^x, \exp(x)$	$e^x, \exp(x)$

Remember that the second "rule" is actually part of the definition of the exponential function.

We can combine these rules to differentiate even a complicated product:

$$h(x) = (e^x + x^3 - 3x + 1)(e^{3x^2} - x)$$

First, we notice that the function is built out of a product:

$$h(x) = \underbrace{(e^x + x^3 - 3x + 1)}_{f(x)}\underbrace{(e^{3x^2} - x)}_{g(x)}$$

Therefore, $h'(x) = f'(x)g(x) + f(x)g'(x)$ by the **product rule**. This reduces us to computing two (hopefully simpler) derivatives: those of $f(x)$ and $g(x)$.

First, $f(x) = e^x + x^3 - 3x + 1$ is a sum of several terms, but the **sum rule** tells us we can differentiate them all separately and add them together. Each individual component is straightforward to handle:

The derivative of e^x is e^x by the **exponential rule**.

The derivative of x^3 is $3x^2$ by the **power rule**.

The derivative of $-3x$ is -3 by the **power rule**.

The derivative of 1 is 0 (it's constant).

Adding these all up according to the **sum rule**, we get

$$f'(x) = e^x + 3x^2 - 3$$

Next up is $g(x)$. We can again use the **sum rule** so that we only have to differentiate e^{3x} and $-x$ separately. The **power rule** tells us right away that the derivative of the second of those is -1.

The derivative of $2e^{3x^2}$ is trickier. Here we will use the **chain rule** to work out the derivative of e^{3x^2}, which is the **composition** of e^x and $3x^2$. If we call those functions $a(x)$ and $b(x)$, respectively, then we seek the derivative of $a(b(x))$, which by **chain rule** should be $a'(b(x))b'(x)$. The derivative of $a(x) = e^x$ is e^x by the **exponential rule** while the derivative of $b(x) = 3x^2$ is $6x$ by the **power rule**. Combining those with the **chain rule**, we obtain

$$a'(b(x))b'(x) = e^{3x^2}6x = 6xe^{3x^2}$$

Combining these two with **sum rule**, we get

$$g'(x) = 6xe^{3x^2} - 1$$

To recap, we have worked out:

$$f'(x) = e^x + 3x^2 - 3 \qquad \text{and} \qquad g'(x) = 6xe^{3x^2} - 1$$

while the quantity we really want is

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

All we have to do is substitute in the quantities $f(x), f'(x), g(x), g'(x)$, which we all know by now!!

$$h'(x) = (e^x + 3x^2 - 3)(e^{3x^2} - x) + (e^x + x^3 - 3x + 1)(6xe^{3x^2} - 1)$$