

1. Suppose you take out a car loan from the bank for the following amounts and interest rates. Write down a rate equation describing the situation, and then an exact solution.

(a) \$12000 at 6%

$$\left. \begin{aligned} B'(t) &= 0.06 B(t) \\ B(0) &= 12000 \end{aligned} \right\} B(t) = 12000 e^{0.06t}$$

(b) \$30000 at 12%

$$\left. \begin{aligned} B'(t) &= 0.12 B(t) \\ B(0) &= 30000 \end{aligned} \right\} B(t) = 30000 e^{0.12t}$$

(c) \$3000 at 20%

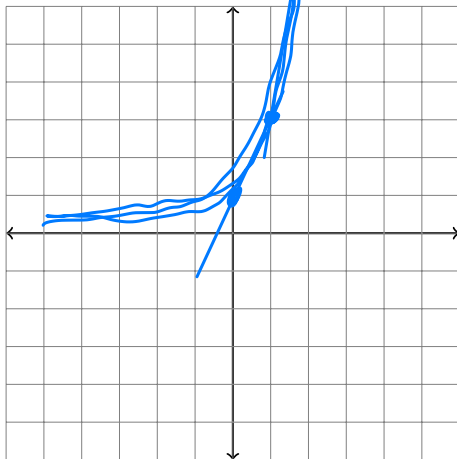
$$\left. \begin{aligned} B'(t) &= 0.2 B(t) \\ B(0) &= 3000 \end{aligned} \right\} B(t) = 3000 e^{0.2t}$$

(d) \$10000 at 2%

$$\left. \begin{aligned} B'(t) &= 0.02 B(t) \\ B(0) &= 10000 \end{aligned} \right\} B(t) = 10000 e^{0.02t}$$

2. Sketch solutions to the following rate equations.

(a) $f'(x) = 2f(x)$ and $f(0) = 1$



x	$f(x)$	$f'(x)$
0	1	2
1	3	6
2	9	18
\vdots	\vdots	\vdots

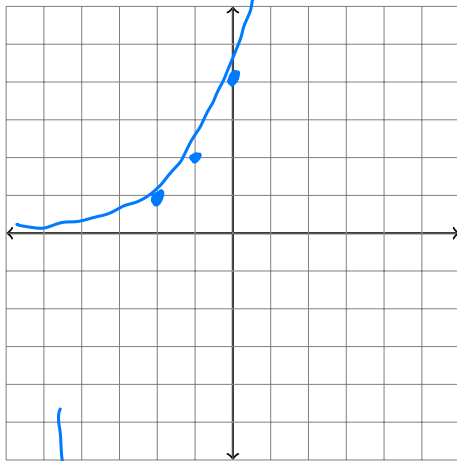
$$f(x) = f(0) + f'(0)x$$

$$f(1) = 1 + 2 \cdot 1 = 3$$

$$f(x) = f(1) + f'(1)(x-1)$$

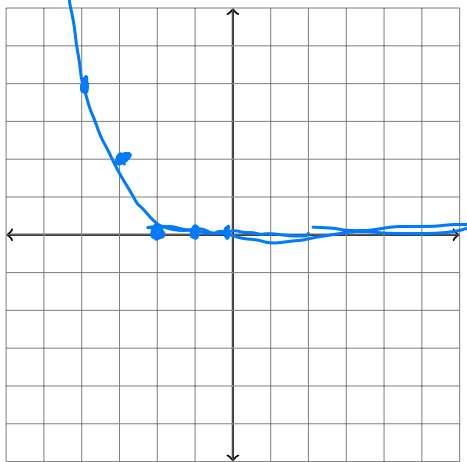
$$f(2) = 3 + 6(2-1) = 9$$

(b) $f'(x) = f(x)$ and $f(-2) = 1$



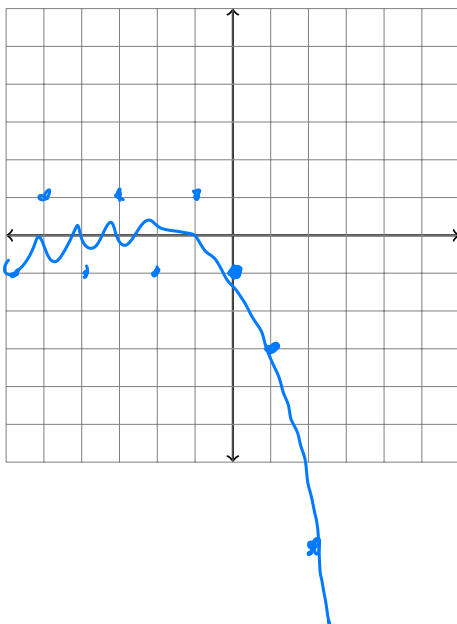
x	$f(x)$	$f'(x)$
-2	1	1
-1	2	2
0	4	4
1	8	8

(c) $f'(x) = -f(x)$ and $f(-3) = 2$



x	$f(x)$	$f'(x)$
-3	2	-2
-2	1	-1
-1	0.5	-0.5

(d) $f'(x) = 2f(x)$ and $f(0) = -1$



x	$f(x)$	$f'(x)$
0	-1	-2
1	-3	-6
2	-9	-18
-1	1	2

3. A mysterious pond algae grows in a waste stabilization pond. When there is more than 10000 algae/liter, the pond will have to be drained and reset. Based on the following information, approximately how often will this process need to be performed?

- over the course of a day, each algae grows and splits into 3 more algae.
- a freshly reset pond contains roughly 0.5 algae per liter.

$$\left. \begin{array}{l} \rightarrow A'(t) = 3A(t) \\ \rightarrow A(0) = 0.5 \end{array} \right\} \begin{array}{l} A(t) = 0.5 e^{3t} \\ A(3) \approx 4000 \\ A(4) \approx 80000 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{a bit} \\ \text{more} \\ \text{then} \\ 3 \text{ days} \end{array}$$

4. Rabbits are known for breeding rapidly: the average litter has 5 babies, and a female rabbit has around 4 litters a year. Suppose 20 rabbits are introduced to a new environment. How long will it take for there to be more than 3 million rabbits?

$$\left. \begin{array}{l} B'(t) = 20 \frac{B(t)}{2} \\ B(0) = 20 \end{array} \right\} \begin{array}{l} B(t) = 20 e^{10t} \\ \text{1-2 years} \end{array}$$

years

5 babies x 4 litters

only female rabbits have babies

5. Certain dangerous proteins, called prions, refold other proteins into the same shape as themselves. Suppose 1 prion takes a day to fold 3 other prions out of ambient proteins. Starting with just 4 prions, how long will it take for there to be at least 20000 prions?

$$\left. \begin{array}{l} P'(t) = 3P(t) \\ P(0) = 4 \end{array} \right\} \begin{array}{l} P(t) = 4 e^{3t} \\ \text{about 3 days} \end{array}$$