1. The following functions can all be written as the product or composition of two simpler functions. Find those simpler functions and express the original in those terms (for instance, $x^2(e^{2x}-3)$ is a product of $f(x)=x^2$ and $g(x)=e^{2x}-3$).

2. Differentiate all of the below functions by applying the product or composition rule to the functions you determined previously.

(a) $(e^{2x})^3 - 3(e^{2x})^2 + 8$

(b) e^{x^2+x+1}

(c) $(x^2 - 9x)(x+1)$

(d) $e^x(x^{50} - 2x + 9)$

(e) $(e^{x+9})^4 + (e^{x+9}) - 11$

(f) $(x^5 - x^2 - 1)(x^2 + 2x + 2)$

(g) e^{3x^3-9x+2}

(h) $e^x e^x$

(i) e^{2x}

3. Suppose a population 2000 carp is introduced to a lake (which previously had now carp). The carp reproduce quite rapidly, having nearly 5 baby carp per year per carp. Write a rate equation describing the ferret population function, then give an exact solution to it.

4. Suppose you are studying a slow-growing bacteria in a lab. You start with just 1 bacterium, which takes 2 days to grow and split, making 1 additional bacterium. Write a rate equation for this situation, and its exact solution.

5. (bonus, but don't skip) Recall the fundamental theorem of the derivative, which says that if $f'(x) = 0$ then $f(x)$ must be a constant function.

Look back at (h) and (i) from Question 1. You might have noticed they have the same derivative. Let's say $f(x) = e^x e^x$ and $g(x) = e^{2x}$. Since they have the same derivative, this means that

$$f'(x) - g'(x) = .0$$

What does the fundamental theorem of the derivative tell you about how f(x) and g(x) are related to each other?