1. The following functions can all be written as the product or composition of two simpler functions. Find those simpler functions and express the original in those terms (for instance, $x^2(e^{2x}-3)$ is a product of $f(x)=x^2$ and $g(x)=e^{2x}-3$).

2. Differentiate all of the below functions by applying the product or composition rule to the functions you determined previously.

(a) $(e^{2x})^3 - 3(e^{2x})^2 + 8$

$f(x) = x^3 - 3x^2 + 8$
$g(x) = e^{2x}$

$f(g(x)) \xrightarrow{\text{comp}} f'(g(x))\, g'(x)$
$= \left(3(e^{2x})^2 - 6(e^{2x})\right)\left(2e^{2x}\right)$

(b) e^{x^2+x+1}

$f(x) = e^x$
$g(x) = x^2+x+1$

$f(g(x)) \rightarrow f'(g(x))\, g'(x)$
$= e^{x^2+x+1}(2x+1)$

(c) $(x^2-9x)(x+1)$

$f(x) \cdot g(x)$

prod

$\hookrightarrow f'(x)g(x) + f(x)g'(x) = (2x-9)(x+1) + (x^2-9x)(1)$

(d) $e^x(x^{50}-2x+9)$

$f(x)$ $g(x)$

$f'(x)g(x) + f(x)g'(x) = e^x(x^{50}-2x+9) + e^x(50x^{49}-2)$

(e) $(e^{x+9})^4 + (e^{x+9}) - 11$

$f(x) = x^4 + x - 11$
$g(x) = e^{x+9}$

$\xrightarrow{\text{comp}} f'(g(x))\, g'(x)$
$= \left(4(e^{x+9})^3 + 1\right) e^{x+9}$

(f) $(x^5-x^2-1)(x^2+2x+2)$

$f(x)$ $g(x)$

$(5x^4-2x)(x^2+2x+2) + (x^5-x^2-1)(2x+2)$

(g) e^{3x^3-9x+2}

comp
$f(x) = e^x$
$g(x) = 3x^3-9x+2$

$f'(g(x))\, g'(x)$
$= e^{3x^3-9x+2}(9x^2-9)$

(h) $e^xe^x = (e^x)^2$

product

$e^xe^x + e^xe^x = 2(e^x)^2$

(i) e^{2x}

comp $f(x) = e^x$
$g(x) = 2x$

$f'(g(x))\, g'(x)$
$= e^{2x} \cdot 2$

3. Suppose a population 2000 carp is introduced to a lake (which previously had now carp). The carp reproduce quite rapidly, having nearly 5 baby carp per year per carp. Write a rate equation describing ~~the fancy population~~ carp function, then give an exact solution to it.

$C'(t) = 5C(t)$

$C(0) = 2000$

$C(t) = 2000e^{5t}$

t in years

4. Suppose you are studying a slow-growing bacteria in a lab. You start with just 1 bacterium, which takes 2 days to grow and split, making 1 additional bacterium. Write a rate equation for this situation, and its exact solution.

$B'(t) = B(t)$

$B(0) = 1$

$B(t) = e^t$

t in 2day units

5. (bonus, but don't skip) Recall the fundamental theorem of the derivative, which says that if $f'(x) = 0$ then $f(x)$ must be a constant function.

Look back at (h) and (i) from Question 1. You might have noticed they have the same derivative. Let's say $f(x) = e^x e^x$ and $g(x) = e^{2x}$. Since they have the same derivative, this means that

$$f'(x) - g'(x) = .0$$

What does the fundamental theorem of the derivative tell you about how f(x) and g(x) are related to each other?