1. Combine the following functions using the exponential rules. Then, differentiate these (simpler) expressions (using composition rule).

(a) $e^x e^{2x} e^{3x} e^{4x}$

(b) $e^{x^2} e^{2x} e^1$

(c) $e^x e^{-x} e^2$

(d) $e^{x-2x^2+x^3} e^{3x^2}$

2. Calculate a degree 4 approximation of e^x. Use this to estimate e^2. Does this seem like a reasonable estimate (e^2 is roughly 7.5)? What could you do to improve the estimate?

3. Using your previous degree 4 approximation of e^x, find a degree 8 approximation of e^{x^2} (hint: plug in x^2!).

4. Suppose we have a mystery function $f(x)$ which is approximated by

$$2 + x + x^2 + 2x^3 + x^4 + \textbf{error}.$$

What is the derivative of $f(x)$? Use the above to approximate the derivative of $f(x)e^x$ at zero.

5. Suppose we have two functions $f(x)$ and $g(x)$ such that

$$f(g(x)) = x$$

Differentiate the above expression. If $f(x) = x^2$ and $g(x) = \sqrt{x}$, what does this tell us about the derivative of $\sqrt{x}$?

6. Using your answer to Question 5, write down a rate equation satisfied by $\sqrt{x+1}$? Using that rate equation, find a degree 2 polynomial approximation to $\sqrt{x+1}$. (this is a very difficult problem!!)