1. Using logarithm rules, write all of the following expressions in terms of $\ln(3)$**,** $\ln(5)$**, and** $\ln(7)$**.**

a) $\ln(5^5 * 3 * 7^8)$ $= 5\ln 5 + \ln 3 + 8\ln 7$

b) $\ln(5 * 7/3^2)$ $= \ln 5 + \ln 7 - 2\ln 3$

c) $\ln(9 * 5 * 5 * 7) - \ln(3 * 5^2)$ $= \cancel{2}\ln 3 + \cancel{2\ln 5} + \ln 7 - \cancel{\ln 3} + \cancel{2\ln 5} = \ln 3 + \ln 7$

d) $\ln(\overset{3*5}{15} * 3^2 * 7)$ $= 3\ln 3 + \ln 5 + \ln 7$

2. Differentiate the following expressions (logarithm rules will probably help you).

a) $\ln(\underbrace{\overset{\text{prod}}{xe^x - x}}_{g(x)})$, $f(x)$

$f'(x) = \frac{1}{x}$ $\quad g'(x) = xe^x + e^x - 1$ $\xrightarrow{\text{comp}}$ $\frac{1}{xe^x - x}(xe^x + e^x - 1)$

b) $\ln((x+1)(x-1)e^x)$

$= \ln(x+1) + \ln(x-1) + \cancel{\ln e}^x$

$\to \frac{1}{x+1} + \frac{1}{x-2} + 1$

c) $\cancel{e}^{\ln(3+x+x^2)}$

$= 3 + x + x^2 \to 1 + 2x$

3. Differentiate the following expressions. Show your work at each step by writing the rule (log, product, composition, etc) that you are using to move to the next step.

a) $\ln((3x)(x^2 + 2x + 1))$ $= \ln 3 + \ln x + \underbrace{\ln(x^2 + 2x + 1)}_{\text{comp}} \to \frac{1}{x} + \frac{2x+2}{x^2+2x+1}$

b) $\ln((x^2 + 3)e^{x^5 - 2x + 1})$

typo ~~c) $e^{\ln(x^2-3)(x^2-3)}$~~

d) $\ln(e^{9x+9-9x^2} e^{3x+2} e^5 (x^2)(x^2 - 1))$

(b) $= \underbrace{\ln(x^2+3)\,\cancel{\ln e}^{(x^5-2x+1)}}_{\text{product}} \to \frac{2x}{x^2+3}(x^5 - 2x + 1) + \ln(x^2+3)(5x^4 - 2)$

(d) $= \cancel{\ln e}^{9x+9-9x^2} + \cancel{\ln e}^{3x+2} + \cancel{\ln e}^5 + 2\ln x + \ln(x^2-1)$

$\to 9 - 18x + 3 + 0 + 2/x + \frac{2x}{x^2-1}$

4. Calculate the derivative of $\ln(e^{3x+1})$ in two ways (a) simplifying first with log rules then differentiating and (b) differentiating directly with composition rule. Check the two are the same.

(a) $\cancel{\ln e}^{3x+1} = 3x+1 \longrightarrow 3$

(b) $f(x)=\ln(x)$ $\quad f'(x)=\frac{1}{x}$

$g(x)=e^{3x+1}$ $\quad g'(x)=3e^{3x+1}$

$\xrightarrow{\text{comp}} \frac{1}{\cancel{e^{3x+1}}} \cdot 3\cancel{e^{3x+1}} = 3$ ✓

5. As in Question 4, calculate the derivative of $\ln(3x^4)$ in two ways (log rules first, or composition directly) and verify they are the same.

$= \ln(3) + 4\ln(x) \rightsquigarrow \frac{4}{x}$

or $f(x)=\ln(x)$ $\quad f'(x)=\frac{1}{x}$

$g(x)=3x^4$ $\quad g'(x)=12x^3$

$\longrightarrow \frac{1}{\cancel{3}x^{\cancel{4}}} \cancel{12}^{4}\cancel{x^3} = \frac{4}{x}$ ✓

6. (challenge problem) Use the fundamental theorem of the derivative to check that the rate equation

$$f'(x) = \frac{1}{x}$$
$$f(1) = 0$$

has only one solution. (hint: if you have another solution $g(x)$, check that $f(x) - g(x)$ is the constant zero).

so $\begin{cases} g'(x) = \frac{1}{x} \\ g(1) = 0 \end{cases}$

Compare $f(x) - g(x)$: want zero

(1) $f'(x) - g'(x) = \frac{1}{x} - \frac{1}{x} = 0$

so $f(x) - g(x) =$ constant by fun. thm. der.

(2) const $= f(1) - g(1) = 0 - 0 = 0$

$\hookrightarrow f(x) - g(x) = 0$