1. Expand the following expressions using logarithm rules, then differentiate.

(a) $\ln(5xe^x(x^2+1))$

$= \ln(5) + \ln(x) + \ln e^x + \ln(x^2+1)$ $\quad \ln e^x \to x$

$\to 0 + \frac{1}{x} + 1 + \frac{1}{x^2+1}\, 2x$ comp

(b) $\ln(e^{5x})$

$= 5x \rightsquigarrow 5$

(c) $e^{\ln(x^3+4x+1)}$

$= x^3+4x+1 \rightsquigarrow 3x^2+4$

(d) $\ln\left((x^2+2)(x^3)(e^{x+2})\right)$

$= \ln(x^2+2) + \ln(x^3) + \cancel{\ln} e^{x+2}$

$\ln(x^3) = 3\ln x$

$\to \frac{1}{x^2+2}\, 2x + 3\frac{1}{x} + 1$

2. Calculate the derivative of $\ln(1+x)$. Write this as a rate equation (don't forget an initial condition!)

$\overset{\text{comp rule}}{=} \frac{1}{1+x}$

$f'(x) = \frac{1}{1+x}$

$f(0) = \ln(1) = 0$

3. Find a degree 2 polynomial approximation to $\ln(1+x)$ by using your rate equation from the previous Question.

$f(x) = A + Bx + Cx^2 + \cdots$ $\quad 0 = f(0) = A$

$f'(x) = B + 2Cx$

$f'(x) = \frac{1}{1+x} \rightsquigarrow (B+2Cx)(1+x) = B + \underbrace{Bx + 2Cx}_{\text{zero}} + \cancel{x^2}$ $\quad 1 = B$

error

$B + 2C = 0 \xrightarrow{B=1} C = -\frac{1}{2}$

$\ln(1+x) = x - \frac{1}{2}x^2 + \cdots$

$= 1 + x + \frac{1}{2}x^2$

4. What is $e^{\ln(1+x)}$? Compose a degree 2 approximation of e^x with your degree 2 approximation of $\ln(1+x)$ above and compare the two.

$= 1 + x$

" $x - \frac{1}{2}x^2$

$$1 + x = e^{\ln(1+x)} \approx 1 + \left(x - \frac{1}{2}x^2\right) + \frac{1}{2}\left(x - \frac{1}{2}x^2\right)^2 = 1 + x + \cancel{\ldots}$$

degree > 2

5. Differentiate the following:

$\ln(x^3 + x^2 + 1)$ $\qquad e^{x^2}\ln(x)$ $\qquad x\ln(x) + e^x$ $\qquad e^{\ln(2x+3)}x^2 + e^x\ln(x)$

$f(x) \quad g(x)$

compose rule

$f'(x) = \frac{1}{x}, \quad g'(x) = 3x^2 + 2x$

$$\frac{1}{x^3 + x^2 + 1}\,3x^2 + 2x$$

$$2xe^{x^2}\ln(x) + e^{x^2}\frac{1}{x}$$

product

$\underbrace{1 + \ln(x)}_{\text{product}} + e^x$

$2x^3 + 3x^2$ product

$$6x^2 + 6x + e^x\ln(x) + e^x\frac{1}{x}$$

6. Could you compute a polynomial approximation to $\ln(x)$? Remember that our polynomial approximations are accurate near zero - what does $\ln(x)$ look like near zero?

No - $\ln(x) \to -\infty$ as $x \to 0$ and no polynomial can approximate that

7. Using the multiple-derivatives method, determine a degree 2 polynomial approximation to $\ln(1+x)$. Compare with Question 3.

$f(x) = A + Bx + Cx^2 + \cancel{\ldots}$

$0 = \ln(1+0) = f(0) = A$

$f'(x) = \frac{1}{1+x}$

$f''(x) = -\frac{1}{(1+x)^2}$

$1 = \frac{1}{0+1} = f'(0) = B$

$-1 = -\frac{1}{(0+1)^2} = f''(0) = 2C$

$\rightsquigarrow C = -1/2$. Same answer!!!