

1. Sketch a graph of $f(x)$ between $x = -3$ and $x = 3$ by using discrete approximation. Does it look familiar?

$$f(0) = 2$$

$$f'(x) = 3x^2 - 4x - 2$$

2. Differentiate the following functions.

(a) $\ln(x^3 + 3x + 2) - e^x$

(b) $e^{\ln x + 2x - 3}$

(c) $(\ln x)^3 - 2(\ln x)^2 + 3$

(d) $\ln(\ln(x))$

(e) $\ln(3x + 2)e^{2x+1}$

3. An important logarithm identity is $\ln(ax) = \ln a + \ln x$. Treat a as a constant. Use the following steps to determine this from the derivative definition of the logarithm:

- (a) Explain how we can apply the Fundamental Theorem of the Derivative to show that $f(x) = g(x)$ by examining their difference, $f(x) - g(x)$.
- (b) Check that $\ln(ax)$ and $\ln a + \ln x$ have a common initial condition (i.e. an x value you can plug in to each one that results in the same value).
Hint: from the definition, we only know $\ln 1 = 0$.
- (c) Calculate and compare the derivatives of $\ln(ax)$ and $\ln a + \ln x$ in the context of (a).

4. Another important logarithm identity is $\ln(x^r) = r \ln x$ for any r (including fractions, decimals, and negative values of r). Apply the process from Question 3 to verify this identity.

5. Apply logarithm identities to the following expressions, and then differentiate them.

(a) $\ln(2^3 * 3^4 * 35)$

(b) $\ln(x^2 e^{3x})$

(c) $\ln((x - 1)(x + 1)x)$

(d) $\ln(\ln(e^{2x(3x+1)}))$

6. Use the logarithm to solve the following equations involving exponential functions.

(a) $3e^{2x+2} = 500$

(b) $1000e^{0.03t} = 2000$

(c) $e^{3x+9} = e^{x^2+3x}$

(d) Suppose you take out a \$15000 loan at a rate of 8% annually, continuously compounded. If you make no payments, how long will it take for your balance to reach \$30000?

(e) Suppose there are 40 geckos on an island which reproduce at a rate of 7 geckos per 2 geckos per year. How long will it take for the number of geckos to double? Triple? Reach 80,000 geckos?