1. Write linear approximations for each of the following functions at $x = 2$.

(a) $\ln(2x + 3)$

(b) e^{x^2+3}

2. For the following functions $f(x)$, let $g(x)$ be an inverse to $f(x)$. Find $f'(x)$ and use this to calculate $g'(x)$ at the given point.

(a) $f(x) = x^3 + 4x - 1$ at $x = 1$

(b) $f(x) = x^2 - 3x + 5$ at $x = 1$

(c) $f(x) = x \ln x$ at $x = e$ (hint: $f(e) = e$)

3. In groups, explain how the to justify the four main derivative rules (additivity, scaling, product, composition) in terms of operations on lines.

4. For the following functions, find a linear approximation $\ell(x)$ to $f(x)$ at $x = 2$. Convince yourself that $f(x) - \ell(x)$ is the error term, and rigorously that the error is much smaller than linear near $x = 2$ (i.e. $error(x) \underset{2}{<<} x - 2$; equivalently $\lim_{x \to 2} error(x)/(x-2) = 0$).

(a) $f(x) = x^3 - 2x^2 + x$

(b) $f(x) = x^2 - x - 1$

(c) $f(x) = (x-2)^4 + 2x + 1$

(d) $f(x) = (x-2)^{10} + 3(x-2)^4 + 5(x-2) + 1$

5. Calculate the following derivatives in two ways (1) apply exponential and identities then differentiate as usual, (2) differentiate by applying composition rule without using identities. Then use logarithm and exponential identities to show that your answers are the same.

(a) $\ln(x^3(x-3))$

(b) $\ln(e^{x^2+2x+1}$

(c) e^{3x^2+9x-3}

(d) $e^{\ln(x)+2x+x^2}$

(e) $(e^{-3x})^3 + 3(e^{-3x})^2 - 4e^{-4x} + 3$