1. The following functions can all be written as the product or composition of two simpler functions. Find those simpler functions and express the original in those terms (for instance, $x^2(e^{2x} - 3)$ is a product of $f(x) = x^2$ and $g(x) = e^{2x} - 3$). Then, differentiate all of the below functions by applying the product or composition rule.

(a) $(e^{2x})^3 - 3(e^{2x})^2 + 8$

(b) e^{x^2+x+1}

(c) $(x^2 - 9x)(x + 1)$

(d) $e^x(x^{50} - 2x + 9)$

(e) $(e^{x+9})^4 + (e^{x+9}) - 11$

(f) $(x^5 - x^2 - 1)(x^2 + 2x + 2)$

(g) e^{3x^3-9x+2}

(h) $e^x e^x$

(i) e^{2x}

2. Suppose a population 2000 carp is introduced to a lake (which previously had now carp). The carp reproduce quite rapidly, having nearly 5 baby carp per year per carp. Write a rate equation describing the ferret population function, then give an exact solution to it.

3. Suppose you are studying a slow-growing bacteria in a lab. You start with just 1 bacterium, which takes 2 days to grow and split, making 1 additional bacterium. Write a rate equation for this situation, and its exact solution.

4. (bonus, but don't skip) Recall the fundamental theorem of the derivative, which says that if $f'(x) = 0$ then $f(x)$ must be a constant function.

Look back at (h) and (i) from Question 1. You might have noticed they have the same derivative. Let's say $f(x) = e^x e^x$ and $g(x) = e^{2x}$. Since they have the same derivative, this means that

$$f'(x) - g'(x) = .0$$

What does the fundamental theorem of the derivative tell you about how $f(x)$ and $g(x)$ are related to each other?

5. Suppose you have investments in the following amounts and return rates. Write down a rate equation describing the situation, and then an exact solution.

(a) \$1500 at 6%

(b) \$2400 at 12%

(c) \$100 at 20%

(d) \$20000 at 2%

6. E. coli quickly grows on room temperature food (particularly meat). When there are more than 10000 bacteria/ounce, the meat must be thrown out. Based on the following information, how long can you leave out some meat?

– every twenty minutes, each e. coli grows and splits into 2 more bacteria.

– a fresh piece of meat has about 200 bacteria/ounce.

7. Determine the derivatives of the following functions.

(a) $f(x) = x^3e^x + 2x^2 + e^x - 1$

(b) $f(x) = e^x + 3e^{2x}$

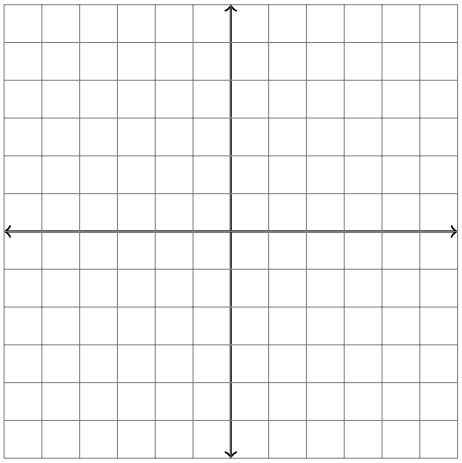
(c) $f(x) = (e^x + 1)(x^2 + 3x - 2)$

(d) $f(x) = xe^x$

8. Sketch a solution to the following differential equation:

$$f'(x) = \frac{1}{4}f(x)$$

$$f(-3) = 1$$



9. Suppose a population 40 ferrets is introduced to a forest. Assume the ferrets produce 1 baby ferret per month per 10 ferrets. Write a rate equation describing the ferret population function, then give an exact solution to it. Using the exact solution and a calculator, estimate when the forest will have more than $100,000$ ferrets.

11. Some modern waste treatments are quite similar to fermentation: bacteria are introduced to waste water in order to consume toxic chemicals.

Suppose we are treating waste water with a kind of bacteria that takes roughly 10 hours to reproduce into 2 more bacteria. A waste pond requires a population of at least $200,000,000,000$ bacteria to function well. If we start with only 2000 bacteria, how long will it take a waste pond to be operable? What if we started with 5000?