1. Differentiate the following polynomial from just the definition of the derivative with Method 1 (undetermined coefficients), first at $x = 2$ and then at general $x = a$, by following the steps below

$$f(x) = x^3 - 4x^2 + 8$$

(a) Write $f(x)$ in linear approximation form.

(b) Write $f(x)$ in the desired form, $A + B(x-a) + C(x-a)^2 + D(x-a)^3$. Why do we go out to degree 3? Circle the error term, and identify which coefficient is the derivative.

(c) Explain why the error term above is much smaller than linear at $x = a$: in particular, rewrite $error(x) <<_a x - a$ in terms of limits then verify that the claim is true.

(d) Expand the expression from (b) and collect terms. Compare coefficients and write down the list of linear equations.

(e) Solve the system of linear equations to determine the derivative. Double-check your answer by differentiating $f(x)$ with the usual power rule, additivity, and scaling "tricks".

2. Suppose there is a meadow with a population of sheep. These sheep reproduce at a rate of 1 baby sheep per year per 2 sheep, and the meadow initially has 20 sheep. By following the steps below, write down an exact formula for $P(t)$ and use a calculator to estimate when the meadow will have 3000 sheep.

(a) The general form of a rate equation for a continuous unrestricted population growth problem is

$$P'(t) = rP(t)$$

$$P(0) = C$$

what are the parameters r and C generally called, and what are their values in this example? **Specify units**.

(b) Explain in words why this rate equation is a reasonable model of population growth.

(c) In class we worked out the exact solution for these rate equations in terms of the exponential function e^x. Use this to write an exact expression for $P(t)$.

(d) With a calculator, plug in some values to your expression above to determine how many years it takes for the meadow to reach 3000 sheep.

Be careful with units

3. Calculate some of the following derivatives. Show your work, specifying which derivative rules you use at each step.

(a) e^{3x+9e^x}

(b) $(2x+2)^3 + 9(2x+2) - 8$

(c) $e^{3x^2+2x-1} + (e^x)^2 + e^x - 4$

(d) $e^{3x-2}e^{x^2-3}e^{x^3}$

(e) $f(g(h(x)+k(x)))$

(f) $e^{2x}(e^{9x-5} + e^{x^2+3}$

(g) $(x^2+1)^2 - e^{x^2+1} + (x^2+1)$